A Discussion of: Job Matching and the Wage Distribution

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Introduction

Introduction

This paper studies a model of learning about static *match quality*.

- These models a number of dynamic facts about the labor market:
  - Wages rise with tenure.
  - Probability of quits initially rise then quickly fall with tenure.
  - Probability of quits fall with current wage.

Main Takeaway: In steady state, a simple model of this type *can* rationalize the cross sectional shape of the wage distribution.

Theoretical Starting Point

Main basis for this paper is Javonavic 1984, adds in Mortensen Pissarides 1994 at the end and shows it doesn't change the implications.

Javonavic 1984, Setup

- Undirected search model with unknown firm-employee match quality.
- At will employment, workers capture entire surplus from match.
- Workers learn their true firm match quality $\mu \sim \mathcal{N}(\bar{\mu}, \sigma_\mu^2)$ over time, given an initial signal $m \sim \mathcal{N}(\mu, \sigma_m^2)$.
- Cumulative output at time t is given by a Weiner Process, i.e. $X(t) \sim \mathcal{N}(\mu t, \sigma^2 t)$, firms update on this.
- Information is destroyed after a match ends because firms and workers never meet again.

Mortensen Pissarides 1994, Setup

- Seeks to explain facts about job creation and destruction over the business cycle.
- Worker match productivity is known, but can change.
- Worker match productivity starts at the *maximum* possible level and is redrawn from a fixed distribution with a Poisson probability once a match is formed.
  - This is what generates job destruction in the model.
- Matching determined by a constant returns to scale matching function $m(v, u)$.
- Workers capture constant fraction of surplus.

Moscarini 2005

The main features needed to get the right shaped wage distribution are:

- Nash Bargaining on wages (which gives a linear sharing rule).
- Binary support of unknown types so $\mu \in \{\mu_L, \mu_H\}$ with $\mu_H > \mu_L$, no initial signal.
- Cumulative output at time t is given by $X(t) \sim \mathcal{N}(\mu t, \sigma^2 t)$ it is observable, and firms and workers Bayesian update using it.
- Unemployed workers and employers meet at a Poisson rate λ, matches are destroyed at a Poisson rate δ.
- *Appropriately "noisy" output.*

This conclusion is unchanged under:

- Undirected on-the-job search.
- Steady state in a GE framework with a constant return to scale matching function $m(v, u)$ and a free entry condition.
  - Unused in calibration.

Key Model Predictions

- If output is given by a sufficiently noisy process, the wage distribution is single peaked with a fat right tail.
- Wages rise with tenure on average.
- The hazard rate of match separation rate initially increases and eventually decreases with tenure.
- Expected future tenure is increasing in the current wage.
- Calibrated model exhibits a kind of unemployment scarring, with welfare

Model Details

Output and Beliefs

- $p_0 = P(\mu = \mu_H) \in (0, 1)$ ex-ante probability of a good match.
- The current belief conditional on the output history is defined to be $p_t \equiv \Pr\left(\mu = \mu_H \mid \mathcal{F}_t^X\right)$

The change in beliefs follows:

$$dp_t = p_t\left(1 - p_t\right) s d\bar{Z}_t \tag{1}$$

For

$$s \equiv \frac{\mu_H - \mu_L}{\sigma} \tag{2}$$

and

$$d\bar{Z}_t \equiv \frac{1}{\sigma}\left[dX_t - p_t \mu_H dt - \left(1 - p_t\right)\mu_L dt\right] \tag{3}$$

Worker HJB Equations

Let worker $W(p)$ be the value of employment with belief p and U be the worker value of unemployment

$$\begin{aligned} rU &= b + \lambda\left[W\left(p_0\right) - U\right] \\ rW(p) &= w(p) + \Sigma(p)W''(p) - \delta[W(p) - U] \end{aligned} \tag{4}$$

Where

$$\Sigma(p) \equiv \frac{1}{2}s^2 p^2 (1-p)^2 \tag{5}$$

is half the variance in the change in posterior beliefs. This measures the speed of learning and governs belief dispersion.

Firm HJB Equations

Let $J(p)$ denote the value of a p-match to a firm and assume the value of a vacancy is 0. Then

$$rJ(p) = \bar{\mu}(p) - w(p) + \Sigma(p)J''(p) - \delta J(p). \quad (6)$$

Where

$$\bar{\mu}(p) \equiv p\mu_H + (1-p)\mu_L. \quad (7)$$

Nash bargaining will imply:

$$\beta J(p) = (1-\beta)[W(p) - U] \quad (8)$$

And that wages are an affine transformation of beliefs:

$$w(p) = (1-\beta)b + \beta\left[\bar{\mu}(p) + \lambda J(p_0)\right] \quad (9)$$

Tenure Function

Match is dissolved if $p_t \leq \underline{p} < p_0$. The tenure function is an increasing concave function of the belief a match is productive.

$$\tau(p) = \frac{1}{\delta}\left\{1 - \left(\frac{p}{\underline{p}}\right)^{1/2-\sqrt{1/4+2\delta/s^2}}\left(\frac{1-p}{1-\underline{p}}\right)^{1/2+\sqrt{1/4+2\delta/s^2}}\right\}.$$

With this result in hand it is fairly easy to show that conditional on match continuation:

1. Wages rise with tenure.
2. The hazard rate of match separations rises initially then declines over time.
3. Expected future tenure is increasing in the current wage.

Distribution of Beliefs and Wages

$$f(p) = \left\{ c_{0f} \left[\left(\frac{1-\underline{p}}{\underline{p}} \frac{p}{1-p} \right)^{\sqrt{1+8\delta/s^2}} - 1 \right] \right. \times \mathbb{I}\left\{ \underline{p} \leq p < p_0 \right\} + c_{1f} \mathbb{I}\left\{ p_0 \leq p \leq 1 \right\} \Big\} \times p^{-1/2-\sqrt{1/4+2\delta/s^2}} (1-p)^{-3/2+\sqrt{1/4+2\delta/s^2}} \tag{10}$$

- The pdf of wages is an affine transformation of this function.
- This function is always increasing to the left of p_0 and decreasing to the right if $\delta \geq s^2$.
- $c_{0f}, c_{1f} > 0$

On the Job Search

Workers now meet firms while on the job at a Poisson rate $\psi\lambda$ where $\psi < 1$.

- When the worker contracts with a new employer the two employers play a poaching auction.
  - In the sub-game perfect equlibria Moscarini considers workers go to the poaching firm if and only if $p < p_0$ and get $W(p_0)$.
  - This will mean that the qualitative properties of the wage distribution remain the same with on the job search.

General Equilibrium

Adds in a CRS matching function $m(v, a) = a^\eta v^{1-\eta}$ with a free entry condition as in Mortensen Pissarides 1994. The job finding rate λ is now given by:

$$\lambda = \frac{m(a, v)}{a} = m\left(1, \frac{v}{a}\right) = \theta^{1-\eta} \tag{11}$$

Where here we have job applicants a instead of unemployed since there is on the job search.

- Problem has a unique stationary solution which features positive employment.
- Doesn't affect qualitative conclusions except that now we "macrofound" λ.

Calibration

Calibration

- Calibrate the model in steady state separately for college educated and non-college educated workers.
- Uses this calibration to graph the distribution of beliefs and tenure function.
- Normalizes out productivity terms μ_i.
- Estimates rate of time preference r and hazard rates λ from the data.
- Select $\delta, \psi, \sigma, b, p_0$ and β to minimize sum of square deviations from empirical moments.
    - Output is normalized so that $\mu_H - \mu_L = 1$, matching will require $\delta \approx \frac{1}{\sigma^2}$

Calibration

Moment	Model	Data
Jobless fraction	9.68	9.5
Fraction who search on the job (%)	5	5
Quits to joblessness	.91	.9
Exogenous separations	1.17	1.2
Job-to-job quits	1.07	1.1
Hires from joblessness	2.08	2.1
(Avgerage - Median)/(SD) of wages	.22	.19
% of wages lost due to displacement	14.3	13.8

Table 1: Calibrated to minimize sum of squared distance between model output and empirical observations.

Model and Observed Job Hazard as Functions of Tenure

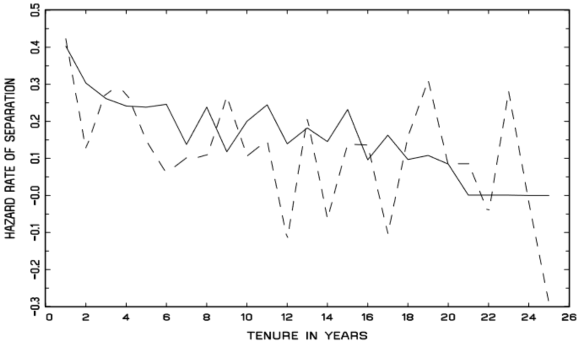


Figure A.1: Hazard rate of separation: model (solid line) and data (dashed line).

Model Wage Distribution

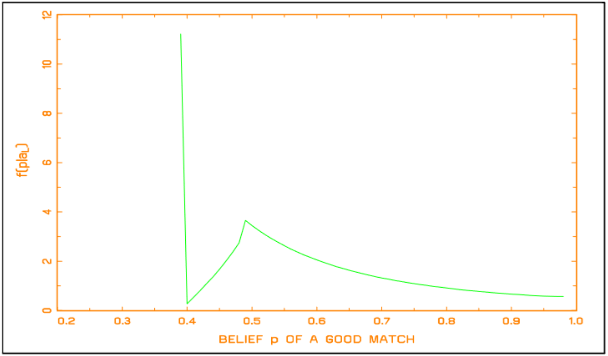
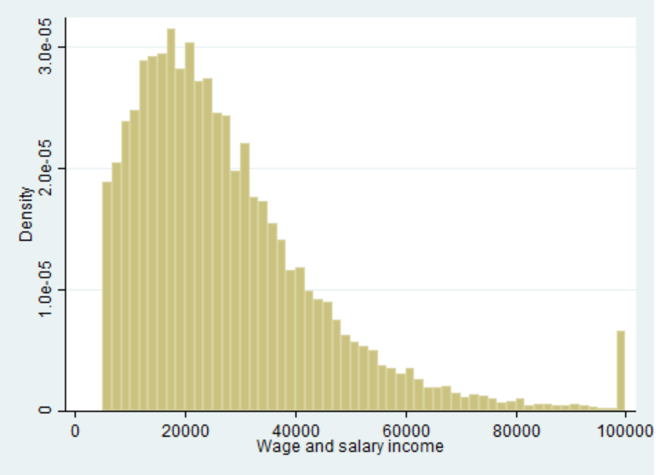


Figure A.3: The ergodic and stationary density of beliefs on match quality for low-skill workers. The atom at the lower bound is the stationary measure of low-skill jobless workers.

Empirical Income Distribution



March CPS post 1996 real wage-income distribution in 1993 dollars for men earning less than 100k a year without a bachelors degree.

Conclusion

Models of learning explain well dynamic facts about wages, this paper shows they also explain the cross section well.

- However, they can only do this when the rate of learning is slow, but not too slow.
    - Suggests a possible test of these models if one can estimate δ and s^2, we should have $s^2 \approx \delta$
- Intuitively, this condition is a requirement that income growth of successful matches looks like random income growth.